

PRE-CALCULUS: by Finney, Demana, Waits and Kennedy
Solving Trigonometric Equations

What you'll Learn About

Solve each trigonometric equation for θ on the interval $[0, 2\pi]$. Then give a formula for all possible angles that could be a solution of the equation.

A) $\sin \theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\frac{\pi}{4} \pm 2\pi k$

$\frac{3\pi}{4} \pm 2\pi k$

$\theta = 45^\circ, 135^\circ$

$45^\circ \pm 360k$

$135^\circ \pm 360k$

B) $\cos \theta = -\frac{1}{2}$

$\theta = 120^\circ, 240^\circ$

$\frac{2\pi}{3}, \frac{4\pi}{3}$

$\theta = 120^\circ \pm 360k$

$240^\circ \pm 360k$

$\frac{2\pi}{3} \pm 2\pi k$

$\frac{4\pi}{3} \pm 2\pi k$

C) $\sin \theta = 1$

$\theta = 90^\circ, \frac{\pi}{2}$

$90^\circ \pm 360k$

$\frac{\pi}{2} \pm 2\pi k$

D) $\cos \theta = 0$

$\theta = 90^\circ, 270^\circ$

$\frac{\pi}{2}, \frac{3\pi}{2}$

$90^\circ \pm 360k$

$270^\circ \pm 360k$

$\frac{\pi}{2} \pm 2\pi k$

$\frac{3\pi}{2} \pm 2\pi k$

F) $\tan \theta = \sqrt{3}$

$60^\circ, 240^\circ$

$60^\circ \pm 360k$

$240^\circ \pm 360k$

$\frac{\pi}{3} \pm \pi k$

$\frac{3\pi}{4} \pm 2\pi k$

$\frac{7\pi}{4} \pm 2\pi k$

$\frac{\pi}{3}, \frac{4\pi}{3}$

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$\{0, 360\}$

Solve each trigonometric equation for θ on the interval $[0, 2\pi]$.

A) $\cos 2\theta = \frac{1}{2}$

$$\begin{aligned} \frac{2\theta}{2} &= \frac{60^\circ}{2} \pm \frac{360k}{2} & \frac{2\theta}{2} &= \frac{300}{2} \pm \frac{360k}{2} \\ \theta &= 30 \pm 180k & \theta &= 150 \pm 180k \\ 30^\circ, 210^\circ & & 150^\circ, 330^\circ & \end{aligned}$$

B) $\sin 3\theta = \frac{1}{2}$

$$\begin{aligned} \frac{3\theta}{3} &= \frac{30}{3} \pm \frac{360k}{3} & \theta &= 10 \pm 120k \\ \theta &= 10 \pm 120k & 10, 130, 250, \\ 50, 170, 290 & & 50 \pm 120k & \end{aligned}$$

C) $\cos \frac{\theta}{3} = \frac{\sqrt{3}}{2}$

$$3\left(\frac{\theta}{3}\right) = (30^\circ \pm 360k)$$

$$\theta = 90 \pm 1080k$$

90

D) $\tan\left(\frac{\theta}{2} + \frac{\pi}{3}\right) = 1$

$\tan\left(\frac{\theta}{2} + 60^\circ\right) = 1$

$$\tan 45^\circ = 1$$

$$\frac{\theta}{2} + 60 = 45^\circ$$

$$\frac{\theta}{2} = -15^\circ$$

$$\theta = -30^\circ$$

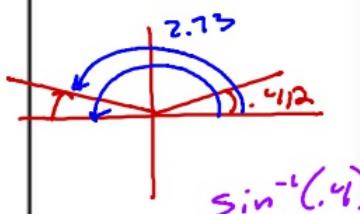
$$\frac{\theta}{2} + 60 = 225^\circ$$

$$\frac{\theta}{2} = 165^\circ$$

F) $\theta = 330^\circ$

$$\cos^{-1}(-.2) = 101.54^\circ$$

$$258.46^\circ$$



E) $\sin \theta = .4$

$$\sin^{-1}(0.4) = 0.412$$

$$\begin{aligned} 3.14 - 0.412 &= 2.73 \\ 23.58^\circ & \\ 180 - 23.58 &= 156.42^\circ \end{aligned}$$

